2018 Fall
CTP431: Music and Audio Computing

Digital Audio Effects

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Introduction

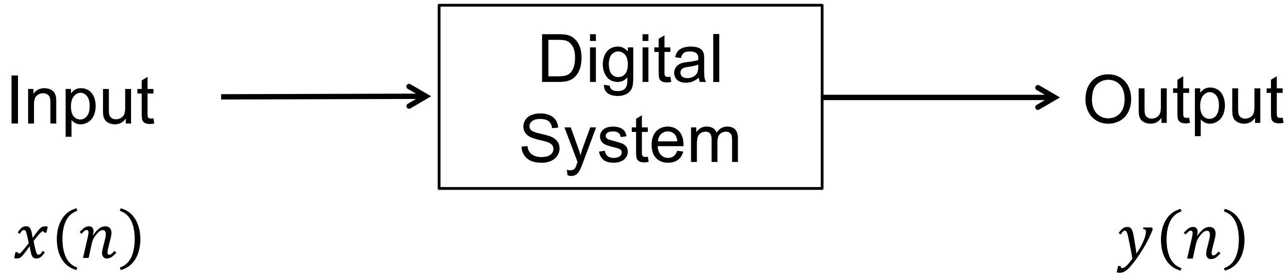
- Amplitude
  - Gain, fade in/out, automation curve, compressor
- Timbre
  - Filters, EQ, distortion, modulation, flanger, vocoder
- Pitch
  - Pitch shifting, transpose
- Time stretching
  - Timing change, tempo adjustment
- Spatial effect
  - Delay, Reverberation, panning, binaural (HRTF)

Source: http://www.uaudio.com/uad/downloads
Source: https://www.izotope.com/en/products/repair-and-edit/rx-post-production-suite.html

Let’s first enjoy some effects!

- http://webaudioplayground.appspot.com

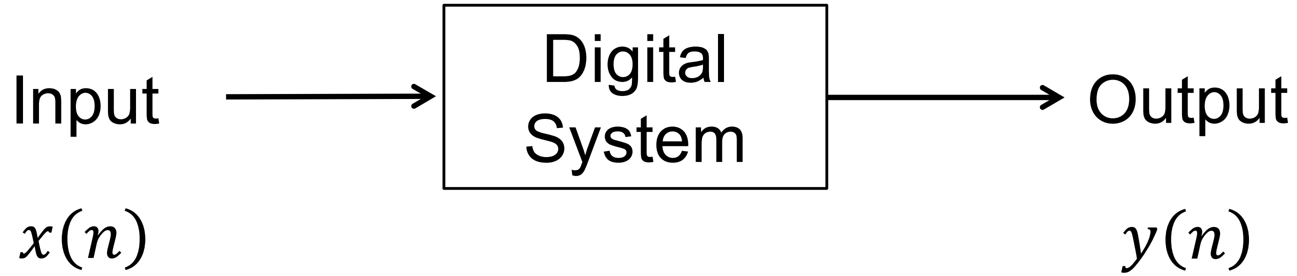
Digital System



- Take the input signal $x(n)$ as a sequence of numbers and returns the output signal $y(n)$ as another sequence of numbers

- We are particularly interested in **linear systems** that are composed of the following operations
  - Multiplication: $y(n) = b_0 \cdot x(n)$
  - Delaying: $y(n) = x(n-1)$
  - Summation: $y(n) = x(n) + x(n-1)$

Linear Time-Invariant (LTI) System



- Linearity
  - Homogeneity: if $x(n) \rightarrow y(n)$, then $\mathrm{a} \cdot x(n) \rightarrow \mathrm{a} \cdot y(n)$
  - Superposition: if $x_1(n) \rightarrow y_1(n)$ and $x_2(n) \rightarrow y_2(n)$, then $x_1(n) + x_2(n) \rightarrow y_1(n) + y_2(n)$

- Time-Invariance
  - If $x(n) \rightarrow y(n)$, then $x(n-N) \rightarrow y(n-N)$ for any N
  - This means that the system does not change its behavior over time

LTI System

- LTI systems in frequency domain
  - No new sinusoidal components are introduced
  - Only existing sinusoids components changes in amplitude and phase.

- Examples of non-LTI systems
  - Clipping
  - Distortion
  - Aliasing
  - Modulation

LTI Digital Filters

- A LTI digital filters performs a combination of the three operations
  - $y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) + \cdots + b_M \cdot x(n-M)$

- This is a general form of **Finite Impulse Response (FIR) filter**

Two Ways of Defining LTI Systems

- By the relation between input $x(n)$ and output $y(n)$
  - Difference equation
  - Signal flow graph

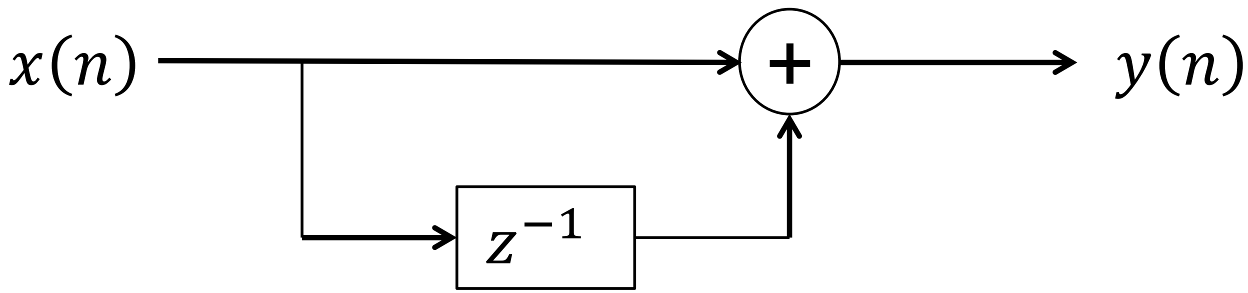
- By the impulse response of the system
  - Measure it by using a unit impulse as input
  - Convolution operation

The Simplest Lowpass Filter

- Difference equation

$$y(n) = x(n) + x(n-1)$$

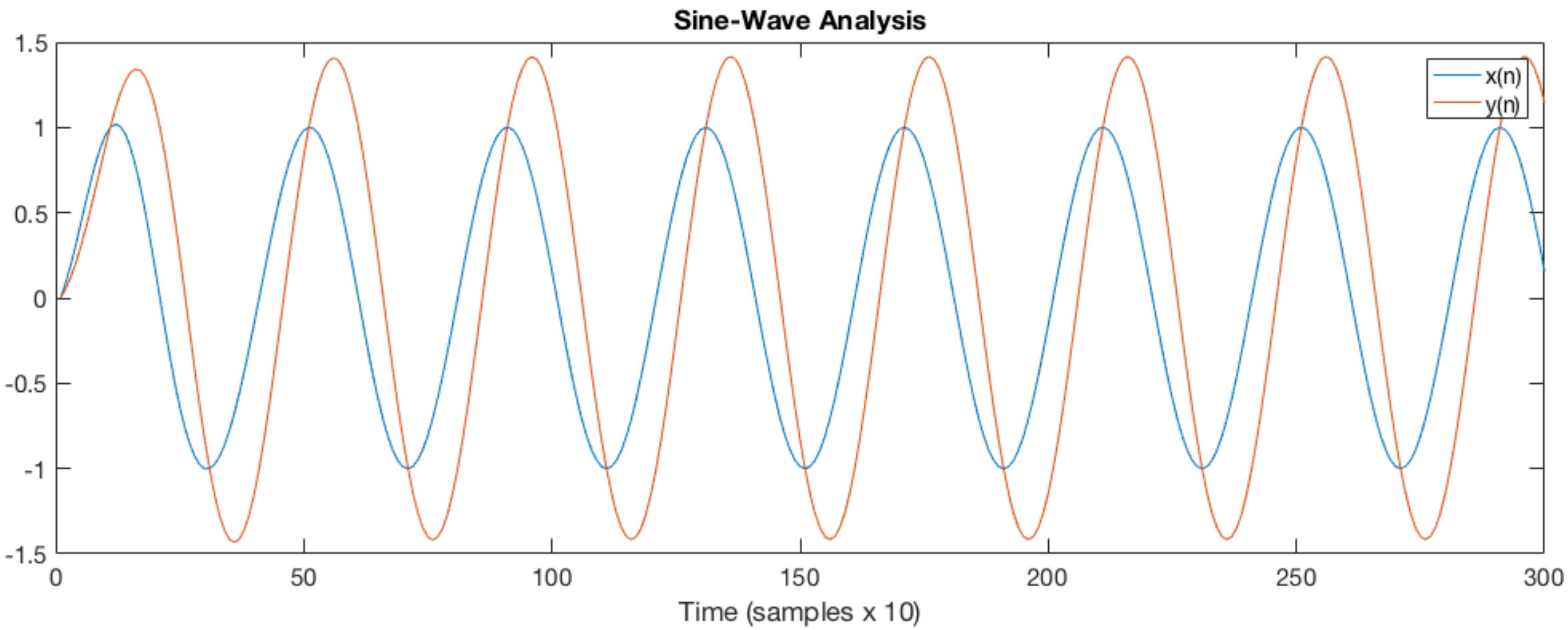
- Signal flow graph



"Delay Operator"

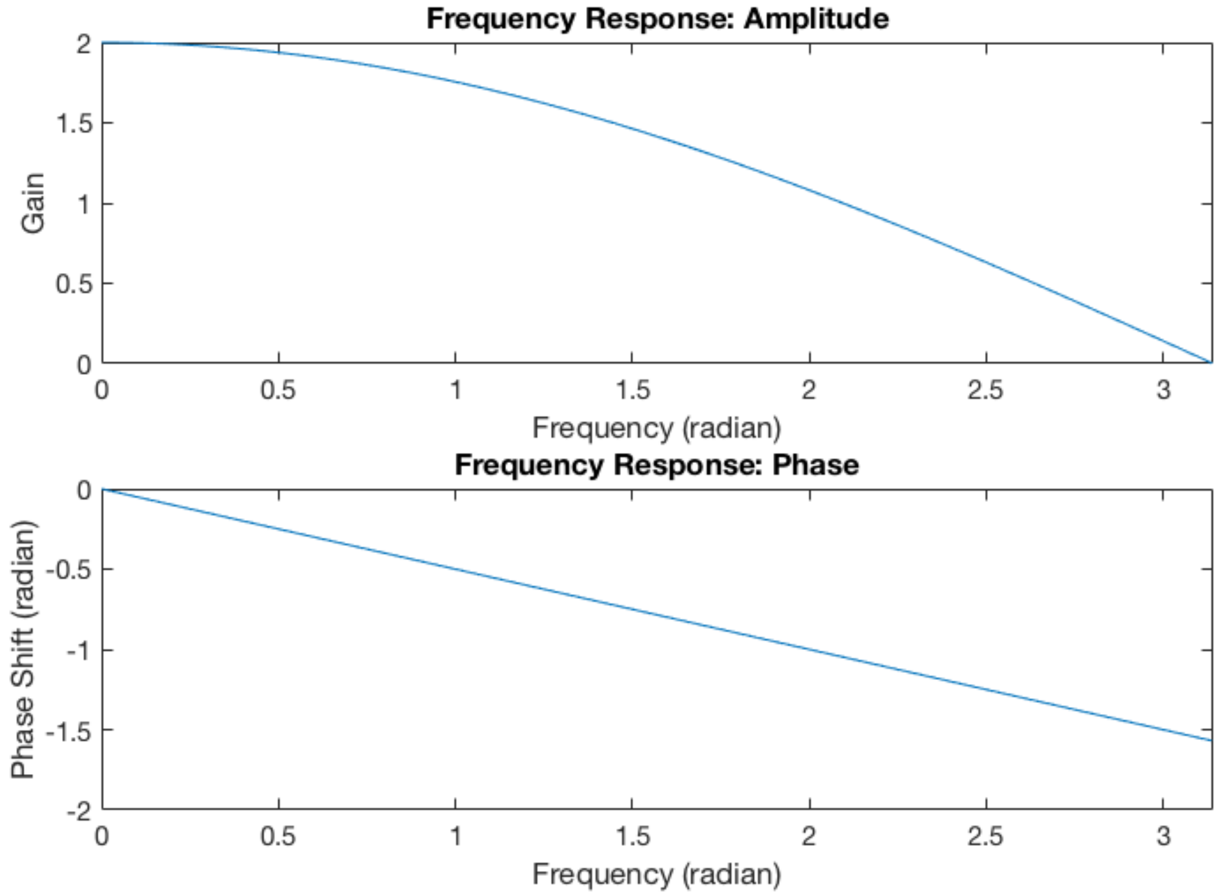
The Simplest Lowpass Filter: Sine-Wave Analysis

- Measure the amplitude and phase changes given a sinusoidal signal input



The Simplest Lowpass Filter: Frequency Response

- Plot the amplitude and phase change over different frequency
  - The frequency sweeps from 0 to the Nyquist rate



The Simplest Lowpass Filter: Frequency Response

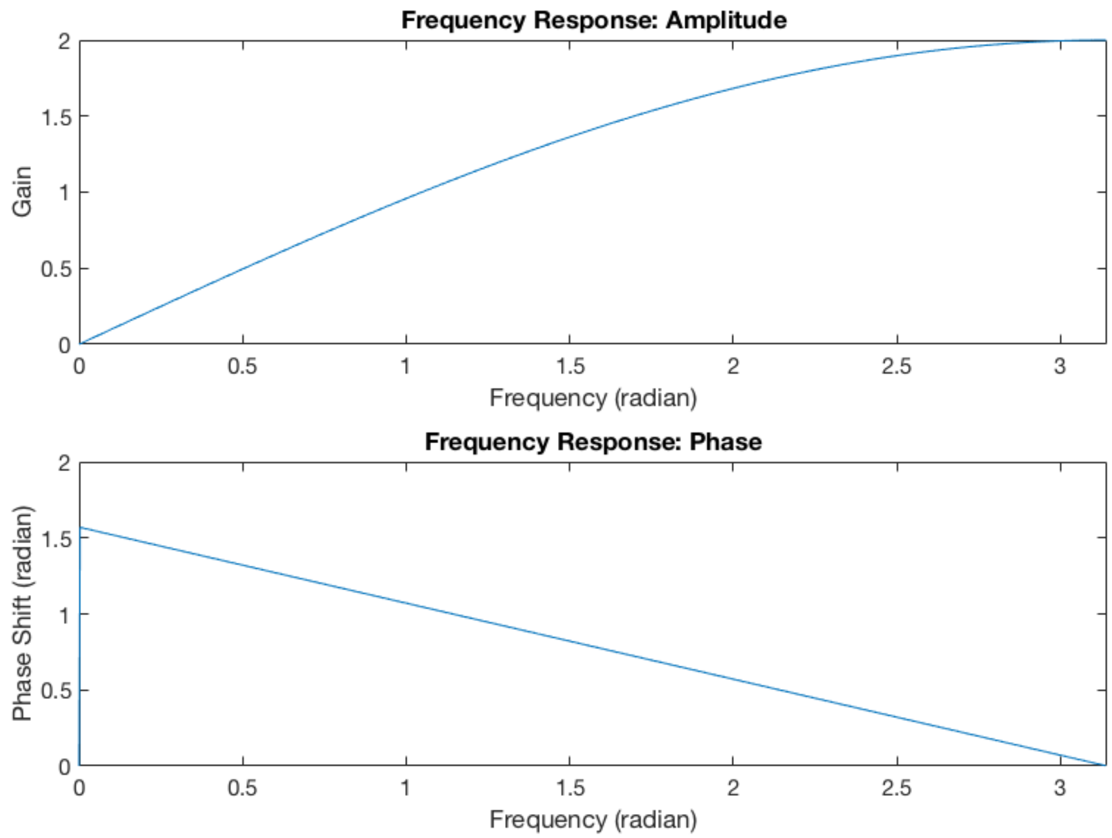
- Mathematical approach
  - Use complex sinusoid as input: $x(n) = e^{j\omega n}$
  - Then, the output is:

$$y(n) = x(n) + x(n-1) = e^{j\omega n} + e^{j\omega(n-1)} = \left(1 + e^{-j\omega}\right) \cdot e^{j\omega n} = \left(1 + e^{-j\omega}\right) \cdot x(n)$$

  - Frequency response: $H(\omega) = \left(1 + e^{-j\omega}\right) = \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right) e^{-j\frac{\omega}{2}} = 2\cos(\frac{\omega}{2})e^{-j\frac{\omega}{2}}$
  - Amplitude response: $|H(\omega)| = 2\cos\left(\frac{\omega}{2}\right)$
  - Phase response: $\angle H(\omega) = -\frac{\omega}{2}$

The Simplest Highpass Filter

- Difference equation: $y(n) = x(n) - x(n-1)$
- Frequency response



Impulse Response

$x(n) = \delta(n)$ $h(n)$ $y(n) = h(n)$

- The filter output when the input is a unit impulse
  - $x(n) = \delta(n) = [1, 0, 0, 0, \dots] \rightarrow y(n) = h(n)$
- Characterizes **the digital system as a sequence of numbers**
  - A system is represented just like audio samples!

Examples: Impulse Response

- The simplest lowpass filter
  - $\text{h}(n) = [1, 1]$

- The simplest highpass filter
  - $\text{h}(n) = [1, -1]$

- Moving-average filter (order=5)
  - $\text{h}(n) = \left[\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right]$

- General FIR Filter
  - $\text{h}(n) = [b_0, b_1, b_2, \dots, b_M]$ → A finite length of impulse response

Convolution

- The output of LTI digital filters is represented by **convolution operation** between $x(n)$ and $h(n)$

$$y(n) = x(n) * h(n) = \sum_{i=0}^{M} x(i) \cdot h(n-i)$$

- Deriving convolution
  - The input can be represented as a time-ordered set of weighted impulses
    - $x(n) = [x_0, x_1, x_2, \dots, x_M] = x_0 \cdot \delta(n) + x_1 \cdot \delta(n-1) + x_2 \cdot \delta(n-2) + \cdots + x_M \cdot \delta(n-M)$
  - By the linearity and time-invariance
    - $y(n) = x_0 \cdot h(n) + x_1 \cdot h(n-1) + x_2 \cdot h(n-2) + \cdots + x_M \cdot h(n-M) = \sum_{i=0}^{M} x(i) \cdot h(n-i)$

Convolution In Practice

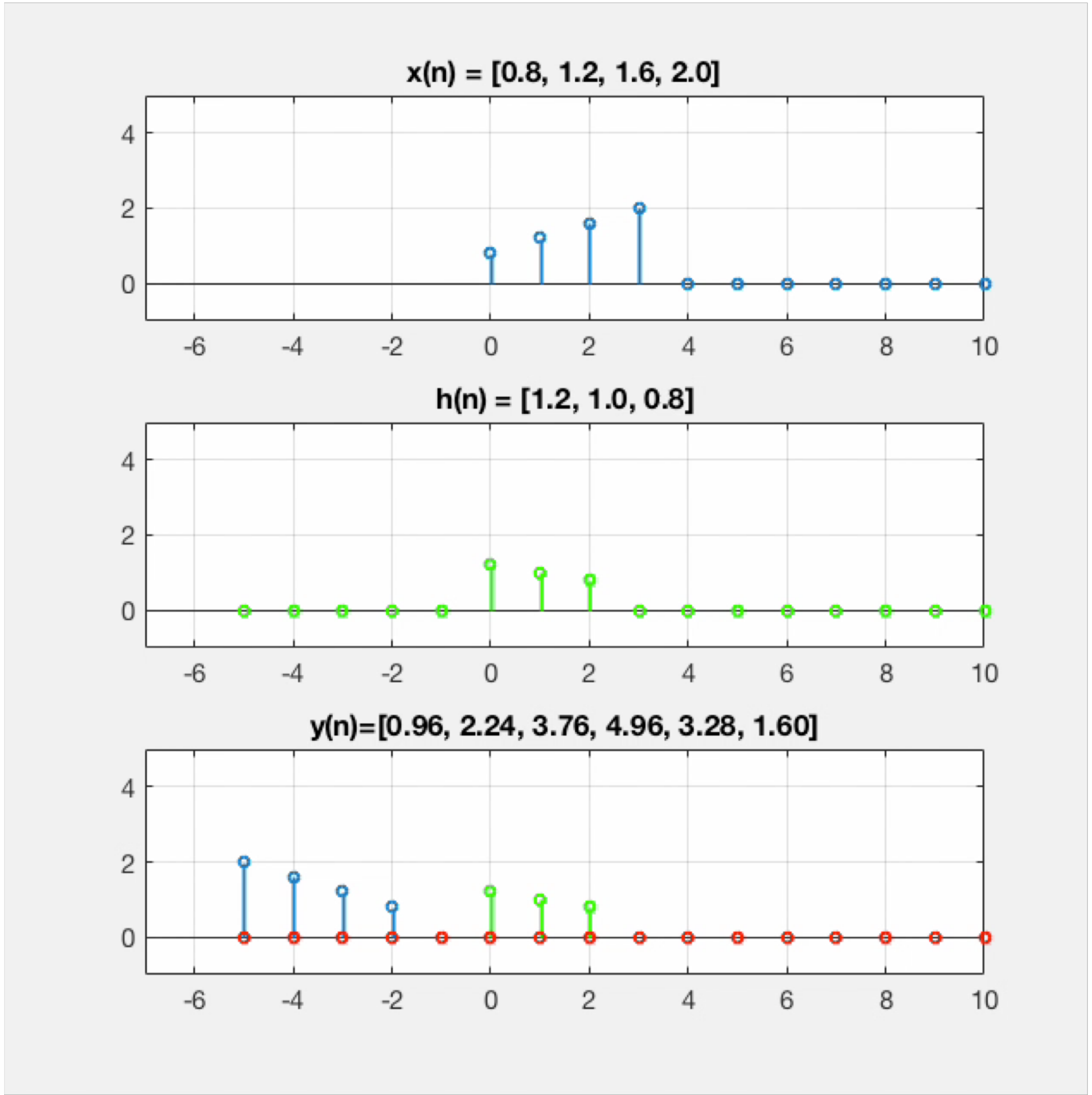
- The practical expression of convolution

$$y(n) = x(n) * h(n) = \sum_{i=0}^{M} x(i) \cdot h(n-i) = \boxed{\sum_{i=0}^{M} h(i) \cdot x(n-i)}$$

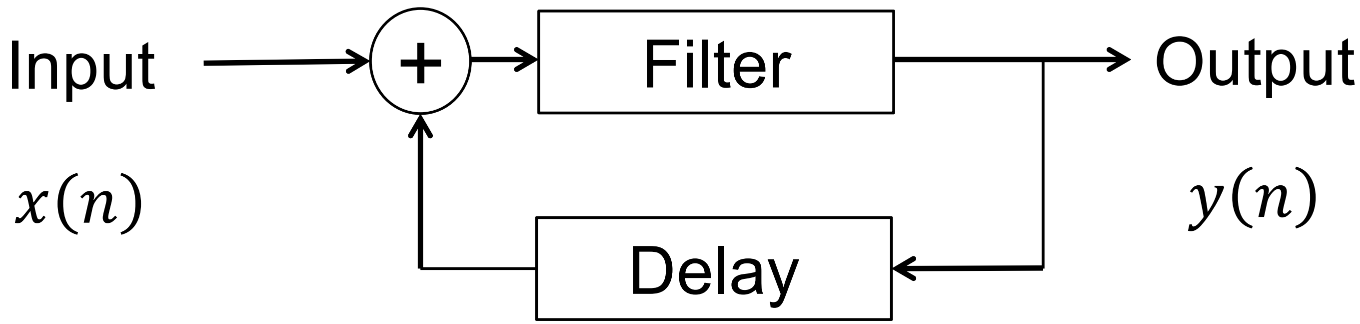
  - This represents input $x(n)$ as a streaming data to the filter $h(n)$

- The length of convolution output
  - If the length of $x(n)$ is M and the length of $h(n)$ is N, the length of $y(n)$is M+N-1

Demo: Convolution



Feedback Filter



- LTI digital filters allow to use the past outputs as input
  - Past outputs: $y(n-1), y(n-2), \dots, y(n-N)$

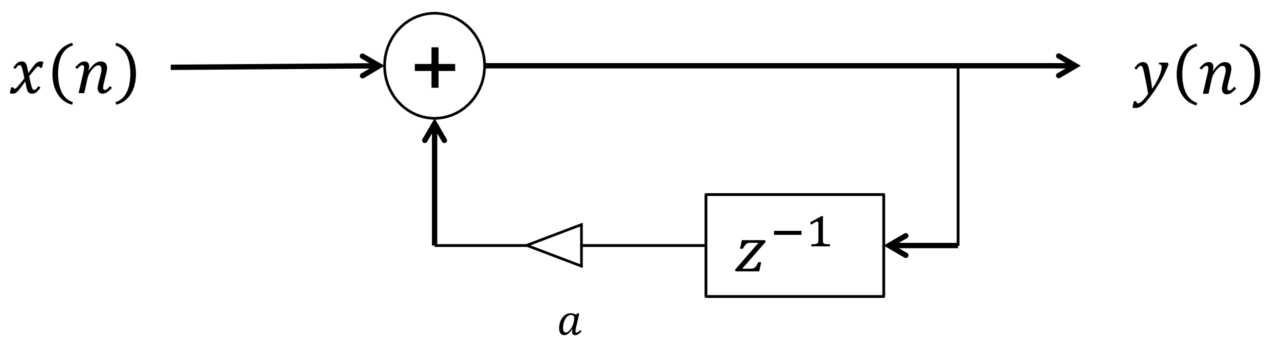
- The whole system can be represented as
  - $y(n) = b_0 \cdot x(n) + a_1 \cdot y(n-1) + a_2 \cdot y(n-2) + \cdots + a_N \cdot y(n-N)$
  - This is a general form of **Infinite Impulse Response (IIR) filter**

A Simple Feedback Lowpass Filter

- Difference equation

$$y(n) = x(n) + a \cdot y(n-1)$$

- Signal flow graph



- When a is slightly less than 1, it is called "Leaky Integrator"

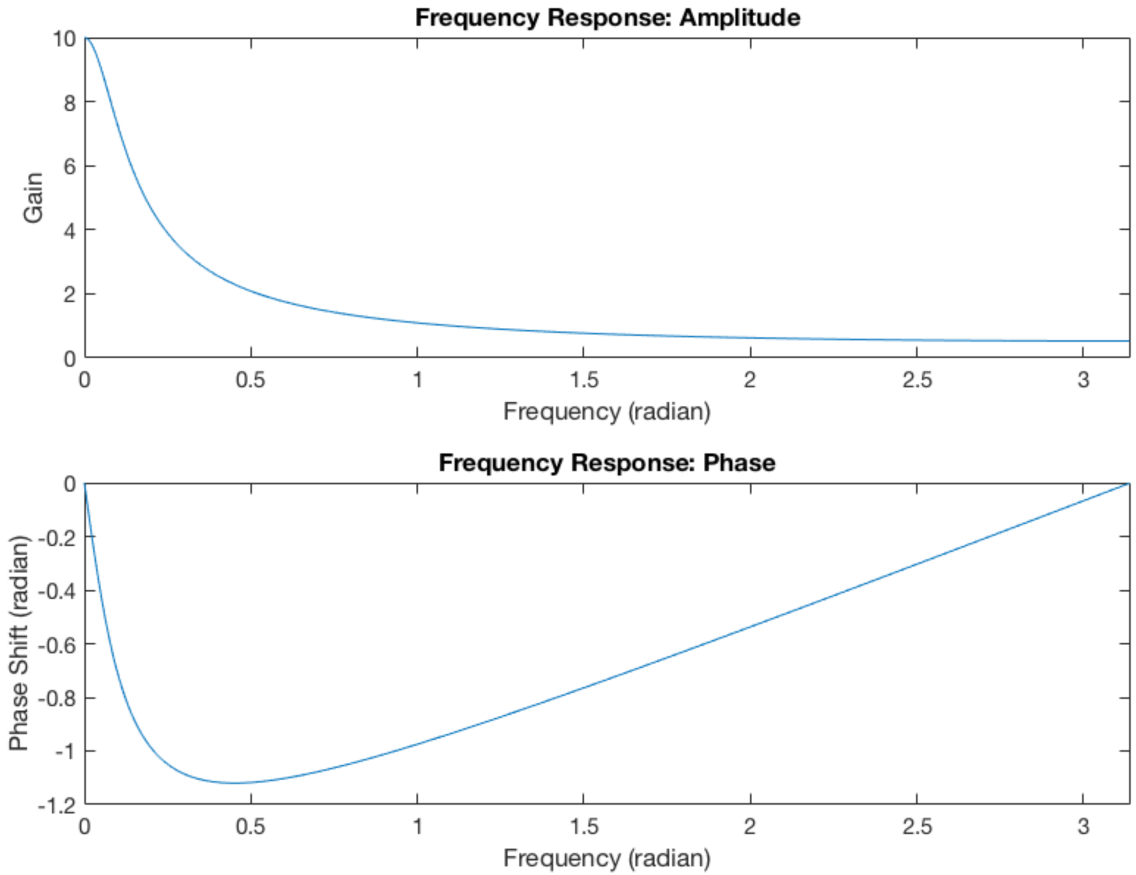
A Simple Feedback Lowpass Filter: Impulse Response

- Impulse response
  - $y(0) = x(0) = 1$
  - $y(1) = x(1) + a \cdot y(0) = a$
  - $y(2) = x(2) + a \cdot y(1) = a^2$
  - $\dots$
  - $y(n) = x(n) + a \cdot y(n-1) = a^n$

- **Stability!**
  - If $a < 1$, the filter output converges (stable)
  - If $a = 1$, the filter output oscillates (critical)
  - If $a > 1$, the filter output diverges (unstable)

A Simple Feedback Lowpass Filter: Frequency Response

- More dramatic change than the simplest lowpass filter (FIR)
  - Phase response is not linear



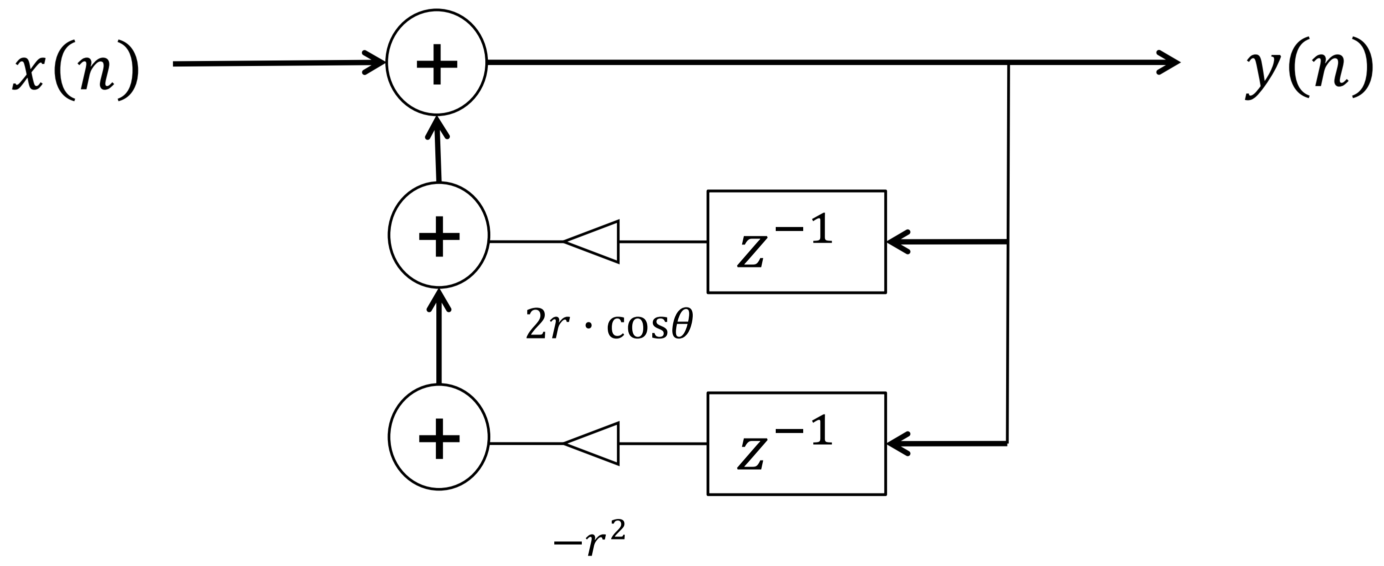
$$y(n) = x(n) + 0.9 \cdot y(n-1)$$

Reson Filter

- Difference equation

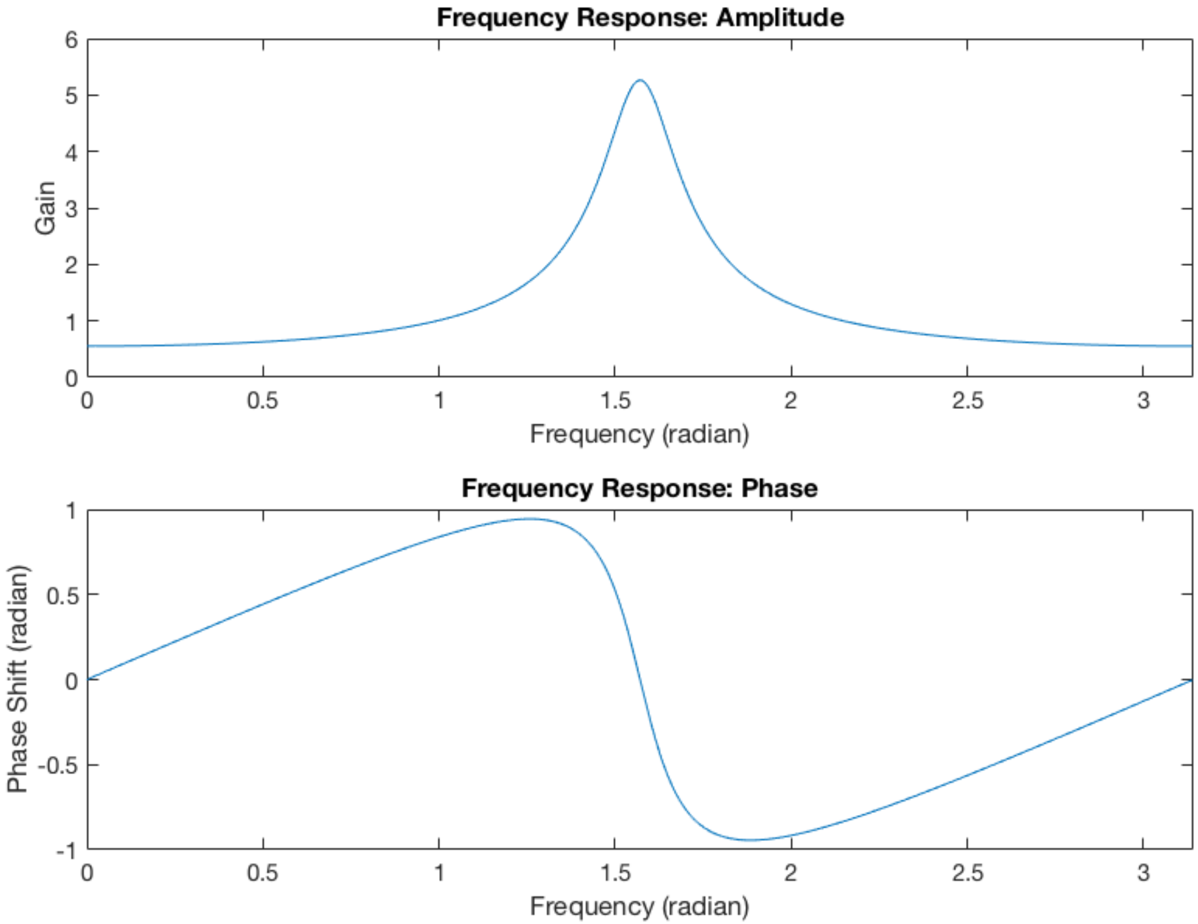
$$y(n) = x(n) + 2r \cdot \cos\theta \cdot y(n-1) - r^2 \cdot y(n-2)$$

- Signal flow graph



Reson Filter: Frequency Response

- Generate resonance at a particular frequency
  - Control the peak height by r and the peak frequency by θ



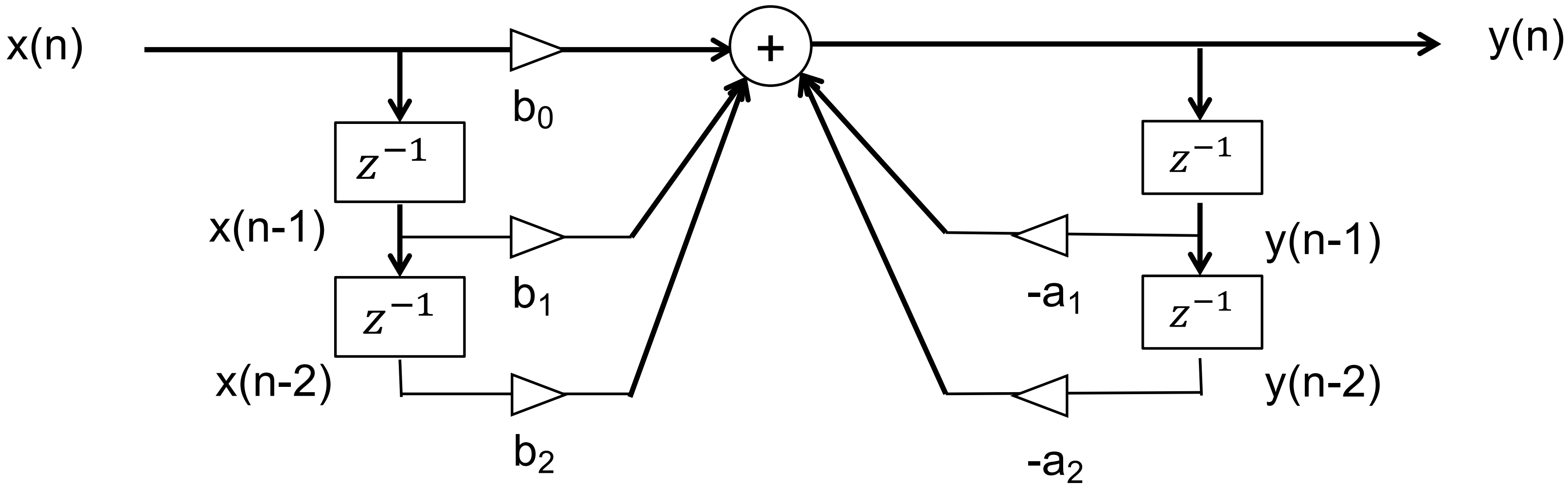
For stability: $r < 1$

Bi-quad Filter

- Difference equation

$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2)$$

- Signal flow graph



Frequency Response

- Sine-wave Analysis
  - $x(n) = e^{j\omega n} \rightarrow x(n-m) = e^{j\omega(n-m)} = e^{-j\omega m}x(n)$ for any m
  - Let's assume that $y(n) = G(\omega)e^{j(\omega n+\theta(\omega))} \rightarrow y(n-m) = e^{-j\omega m}y(n)$ for any m

- Putting this into the different equation

$$y(n) = b_0 \cdot x(n) + b_1 \cdot e^{-j\omega} \cdot x(n) + b_2 \cdot e^{-j2\omega} \cdot x(n) - a_1 \cdot e^{-j\omega} \cdot y(n) - a_2 \cdot e^{-j2\omega} \cdot y(n)$$

$$y(n) = \frac{b_0 + b_1 \cdot e^{-j\omega} + b_2 \cdot e^{-j2\omega}}{1 + a_1 \cdot e^{-j\omega} + a_2 \cdot e^{-j2\omega}} x(n)$$

$$H(\omega) = \frac{b_0 + b_1 \cdot e^{-j\omega} + b_2 \cdot e^{-j2\omega}}{1 + a_1 \cdot e^{-j\omega} + a_2 \cdot e^{-j2\omega}}$$

$H(\omega)$: frequency response
$G(\omega) = |H(\omega)|$: amplitude response
$\theta(\omega) = \angle H(\omega)$: phase response

Z-Transform

- Z-transform
  - Define z to be a variable in complex plane: we call it z-plane
  - When $z = e^{j\omega}$ (on unit circle), the frequency response is a particular case of the following form

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}}$$

  - We call this z-transform or the transfer function of the filter
  - z^{-1} corresponds to one sample delay: delay operator or delay element
  - Filters are often expressed as z-transform: polynomial of z^{-1}

Practical Filters

- One-pole one-zero filters
  - Leaky integrator
  - Moving average
  - DC-removal filters
  - Bass / treble shelving filter

$$H(z) = \frac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1}}$$

- Biquad filters
  - Reson filter
  - Band-pass / notch filters
  - Equalizer

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

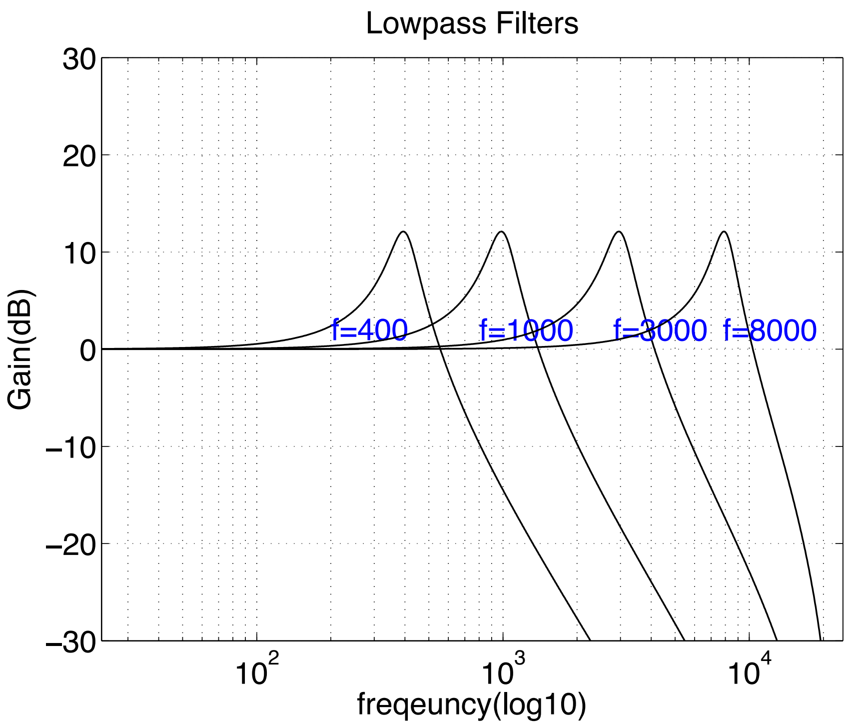
- Any high-order filter can be factored into a combination of one-pole one-zero filters or bi-quad filters!

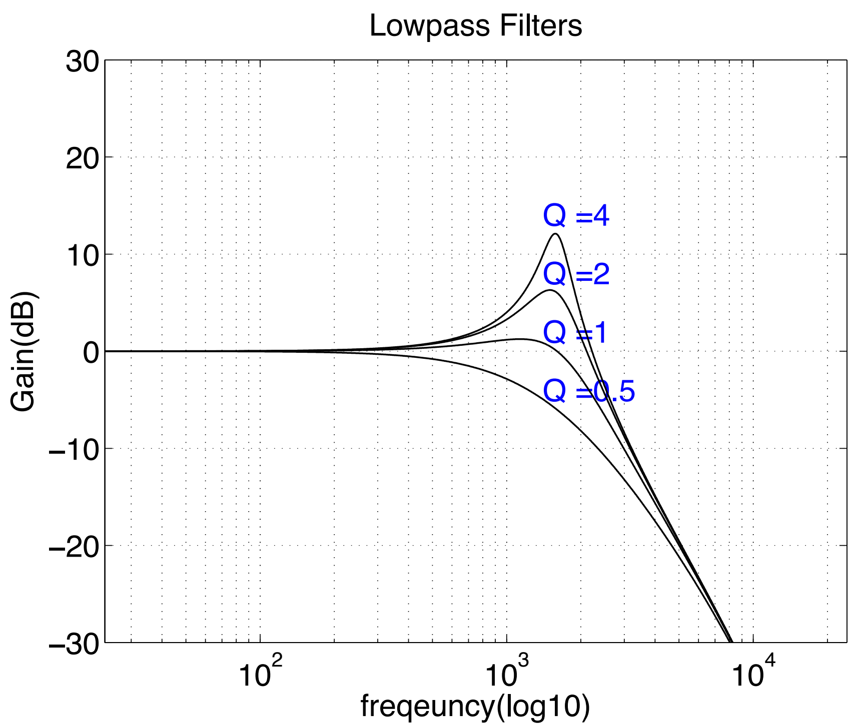
Low-pass Filter

- Transfer Function

$$H(z) = (\frac{1-\cos\Theta}{2})\frac{1+2z^{-1}+1z^{-2}}{(1+\alpha)-2\cos\Theta z^{-1}+(1-\alpha)z^{-2}} \qquad \alpha = \frac{\sin\Theta}{2Q} \qquad \Theta = 2\pi f_c / f_s$$

- fc : cut-off frequency, Q: resonance

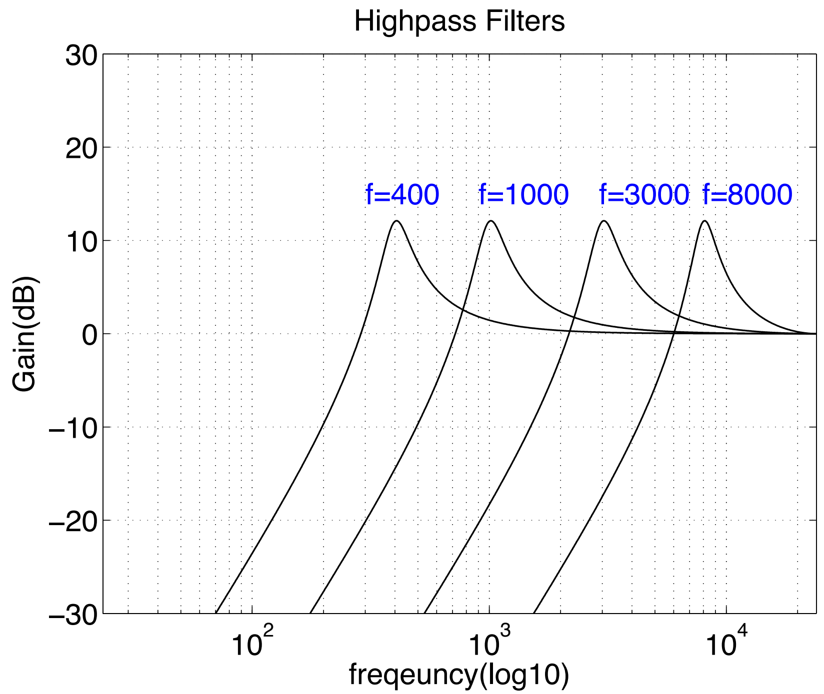


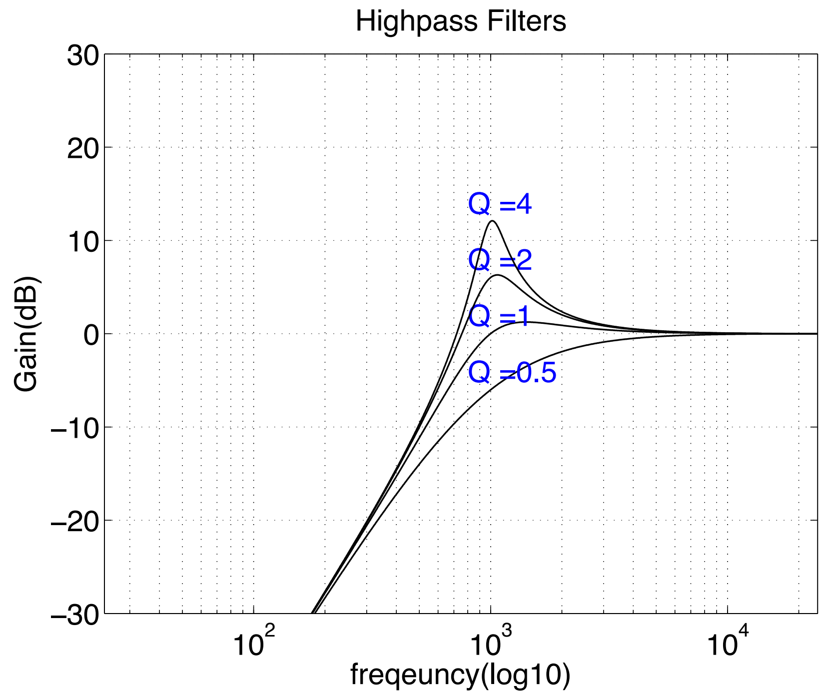


High-pass Filter

- Transfer Function

$$H(z) = \left(\frac{1+\cos\Theta}{2}\right)\frac{1-2z^{-1}+1z^{-2}}{(1+\alpha)-2\cos\Theta z^{-1}+(1-\alpha)z^{-2}} \qquad \alpha = \frac{\sin\Theta}{2Q} \qquad \Theta = 2\pi f_c / f_s$$

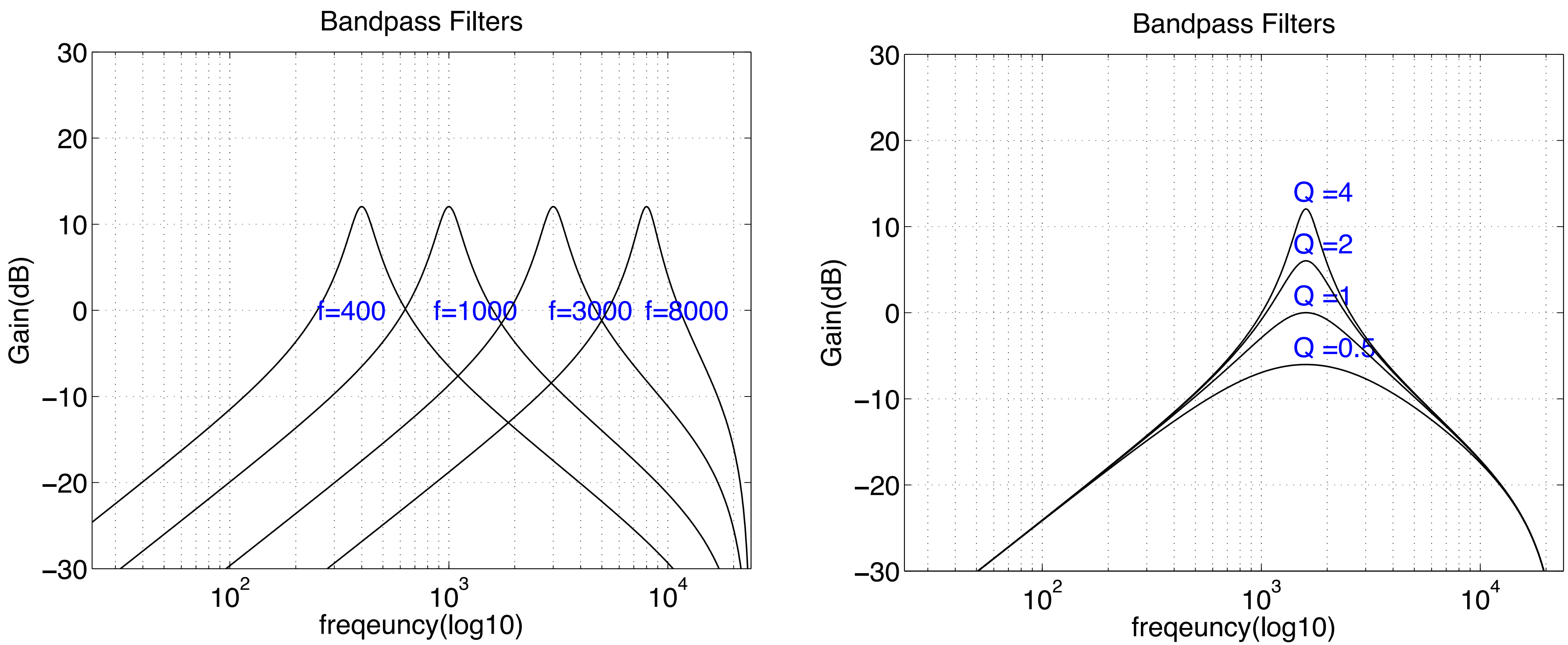




Band-pass filter

- Transfer Function

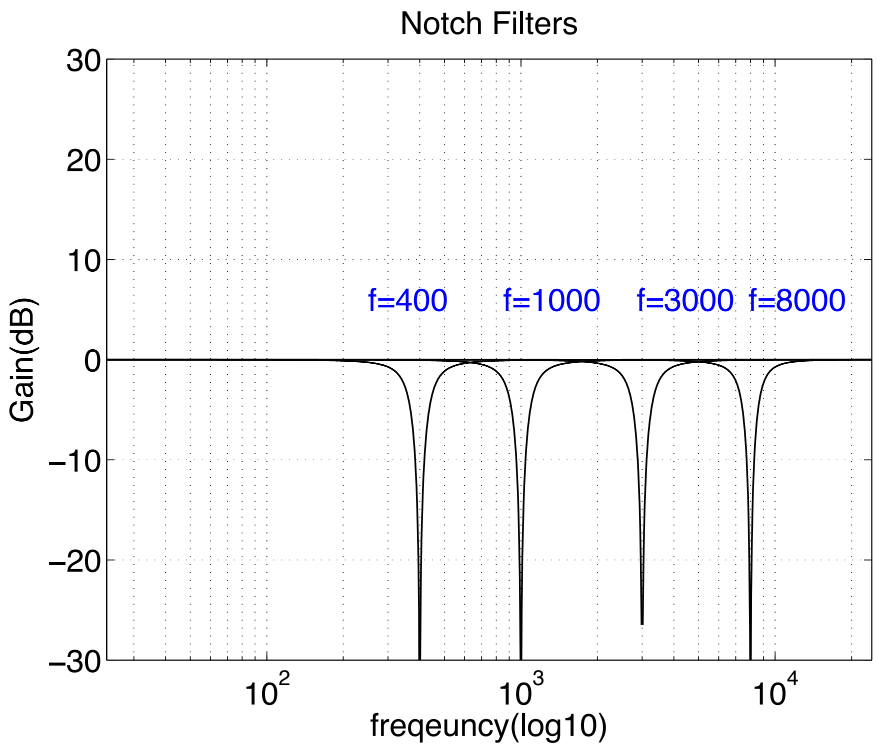
$$H(z) = \left(\frac{\sin\Theta}{2}\right)\frac{1 - z^{-2}}{(1+\alpha) - 2\cos\Theta z^{-1} + (1-\alpha)z^{-2}} \qquad \alpha = \frac{\sin\Theta}{2Q} \qquad \Theta = 2\pi f_c / f_s$$

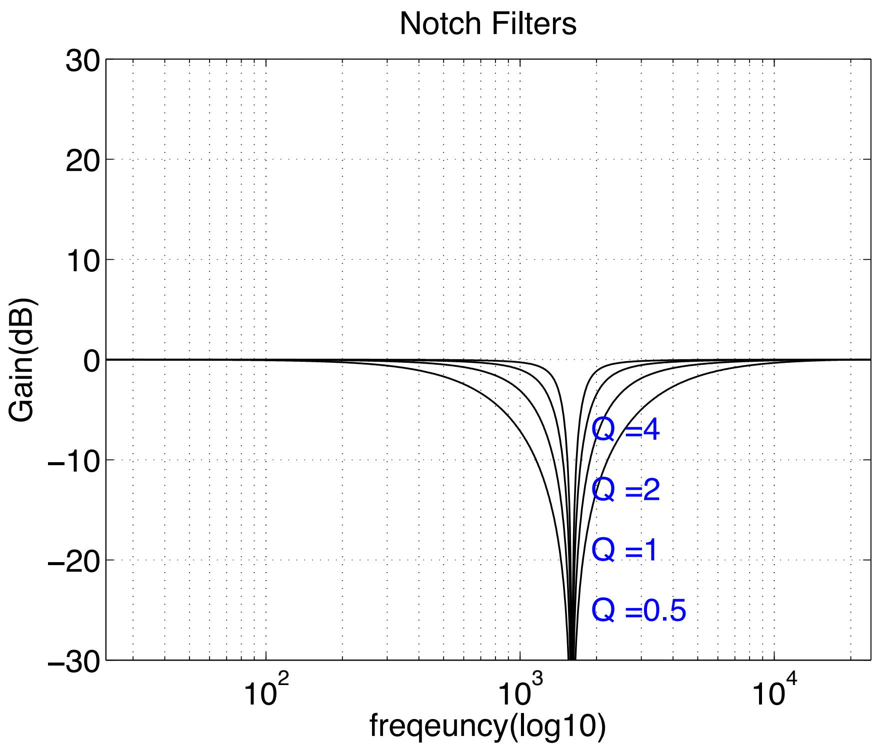


Notch filter

- Transfer Function

$$H(z) = \frac{1 - 2\cos\Theta z^{-1} + z^{-2}}{(1+\alpha) - 2\cos\Theta z^{-1} + (1-\alpha)z^{-2}} \qquad \alpha = \frac{\sin\Theta}{2Q} \qquad \Theta = 2\pi f_c / f_s$$

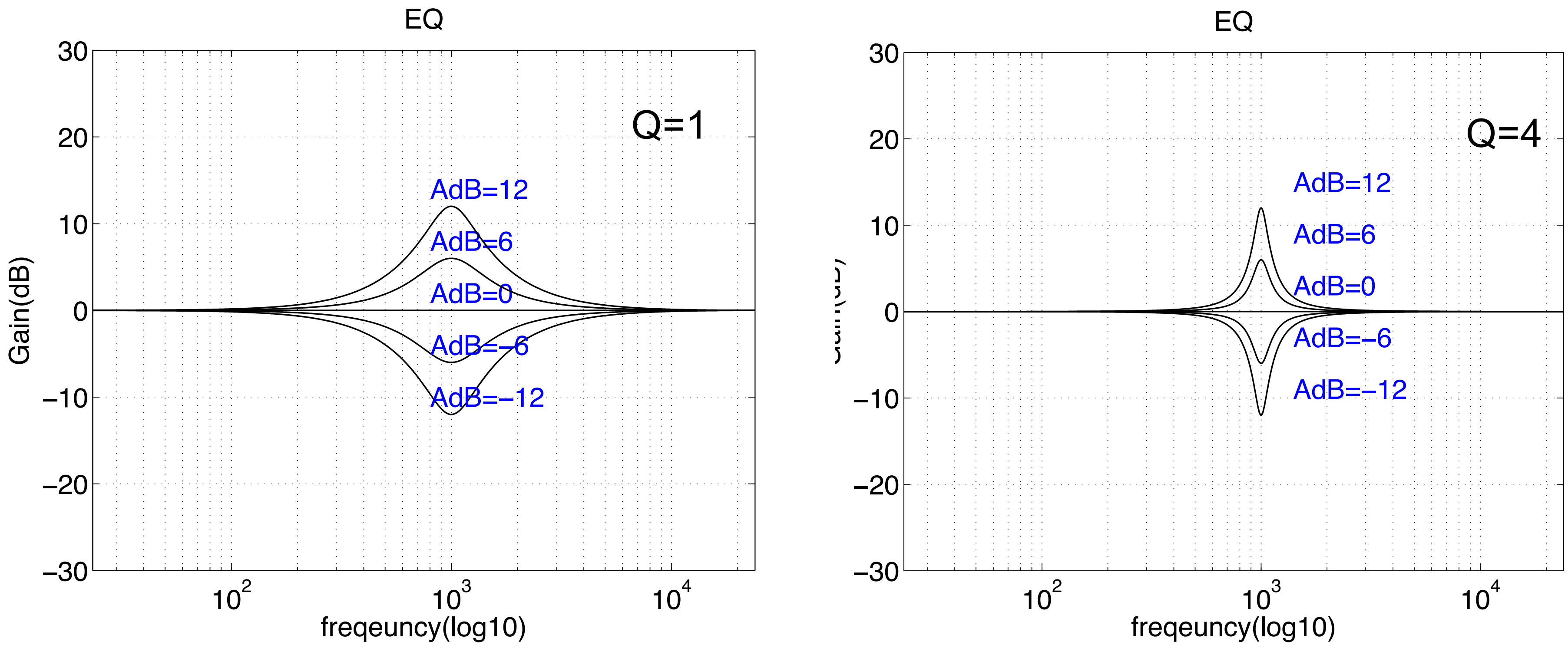




Equalizer

- Transfer Function

$$H(z) = \frac{(1+\alpha \cdot A) - 2\cos\Theta z^{-1} + (1+\alpha \cdot A) z^{-2}}{(1+\alpha / A) - 2\cos\Theta z^{-1} + (1-\alpha / A) z^{-2}} \qquad \alpha = \frac{\sin\Theta}{2Q} \qquad \Theta = 2\pi f_c / f_s$$

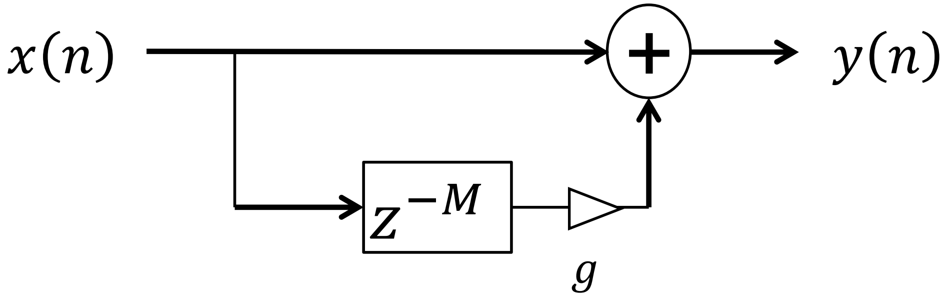


Delay-based Audio Effects

- Types of delay-based audio effect
  - Delay
  - Chorus
  - Flanger
  - Reverberation

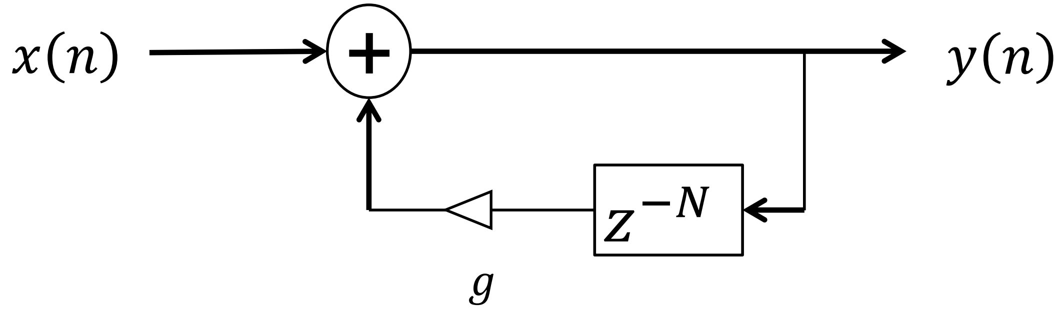
https://www.youtube.com/watch?v=zmN7fK3fKUE&list=PL081D4BE59AE08F99

Comb Filter



$$y(n) = x(n) + g \cdot x(n - M)$$

FIR Comb Filter



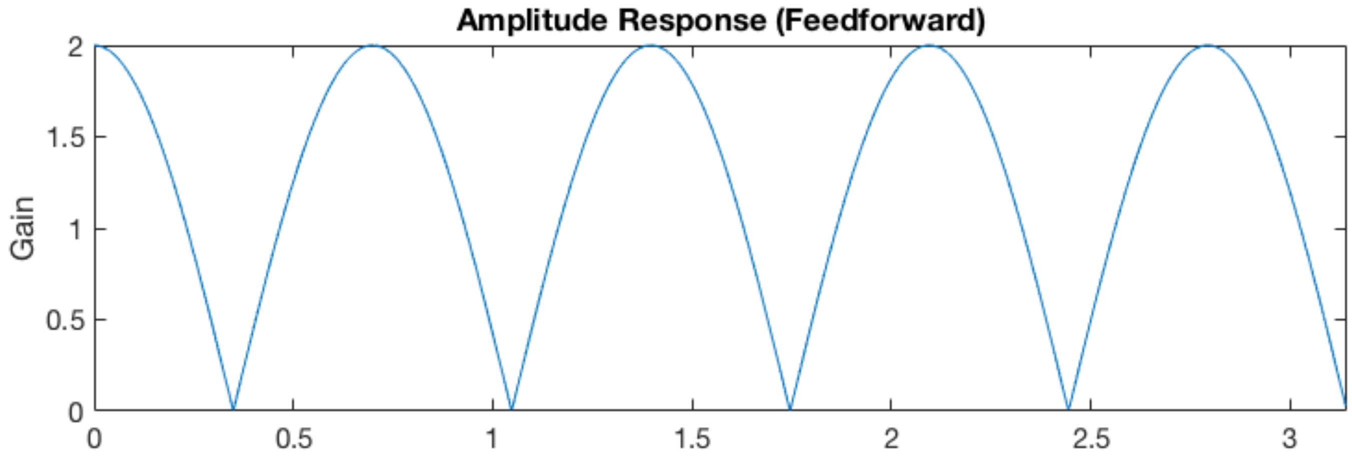
$$y(n) = x(n) + g \cdot y(n - N)$$

IIR Comb FIlters

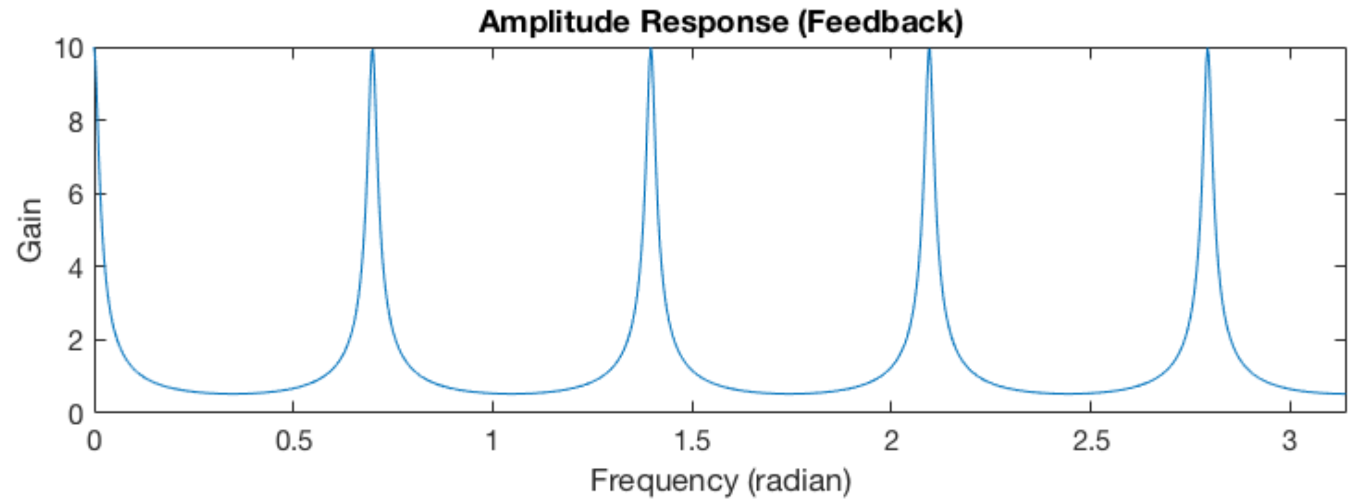
- Implemented by circular buffer: move read and write pointers instead of shift all samples in the delayline

Comb Filter: Frequency Response

- ”Combs” become shaper in the feedback type



$$y(n) = x(n) + x(n-8)$$



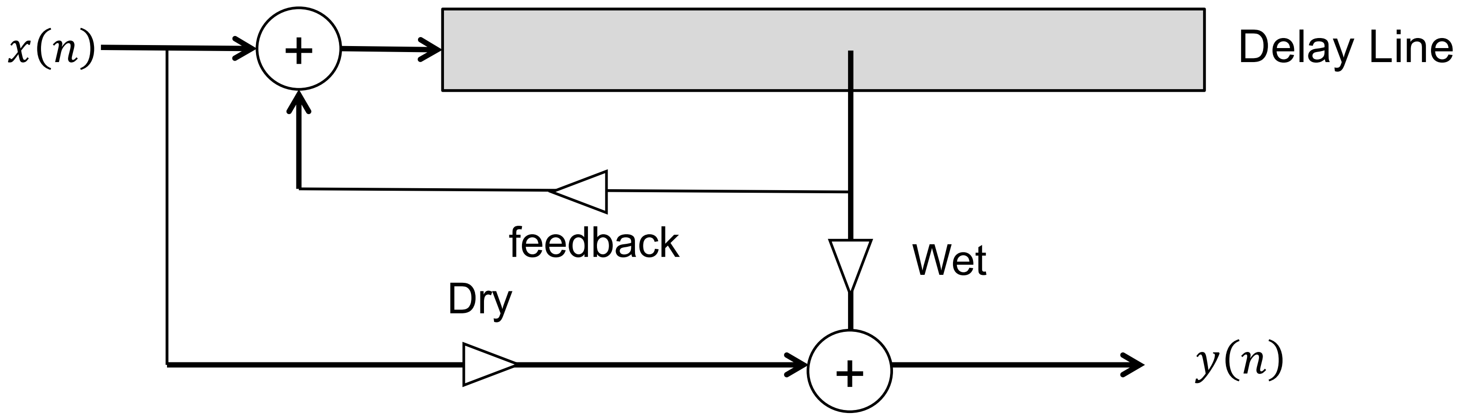
$$y(n) = x(n) + 0.9 \cdot y(n-8)$$

Perception of Time Delay

- The 30 Hz transition
  - Given a repeated click sound (e.g. impulse train):
    - If the rate is less than 30Hz, they are perceived as discrete events.
    - As the rate is above 30 Hz, they are perceive as a tone
  - Demo: http://auditoryneuroscience.com/?q=pitch/click_train

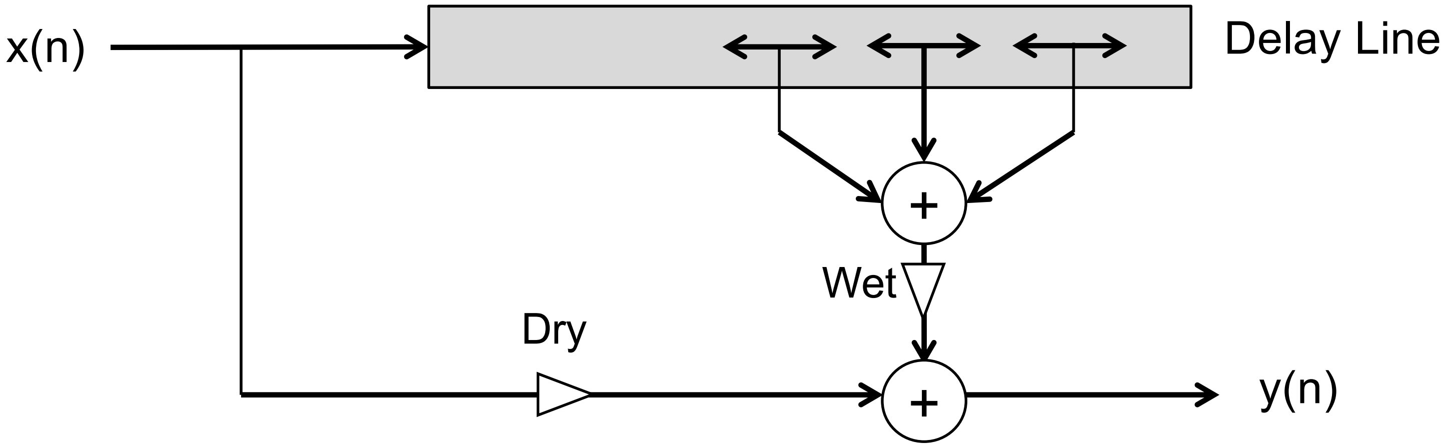
- Feedback comb filter: $y(n) = x(n) + a \cdot y(n - N)$
  - If $\mathrm{N} < \frac{F_s}{30}$ (F_s: sampling rate): models sound propagation and reflection with energy loss on a string (**Karplus-strong model**)
  - If $\mathrm{N} > \frac{F_s}{30}$ (F_s: sampling rate): generate a looped delay

Delay Effect



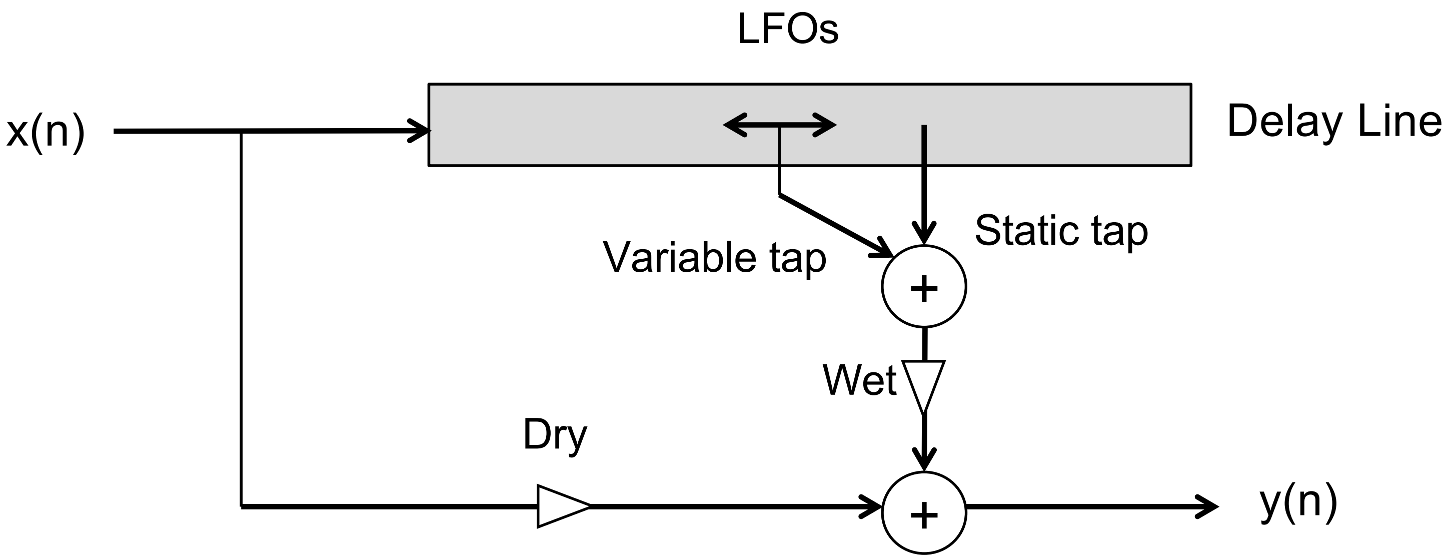
- Generate repetitive loop delay
  - Parameters
    - Feedback gain
    - Delay length
  - Ping-pong delay: cross feedback between left and right channels in stereo
  - The delay length is often synchronized with music tempo

Chorus Effect



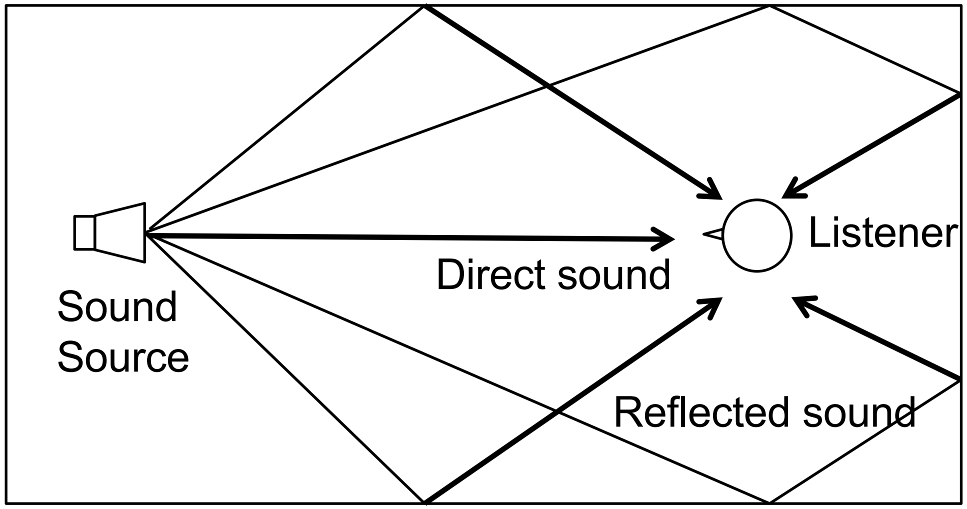
- Gives the illusion of multiple voices playing in unison
  - By summing detuned copies of the input
  - Low frequency oscillators (LFOs) are used to modulate the position of output tops
    - This causes pitch-shift

Flanger Effect



- Emulated by summing one static tap and variable tap in the delay line
  - "Rocket sound"
  - Feed-forward comb filter where harmonic notches vary over frequency.
  - LFO is often synchronized with music tempo

Reverberation



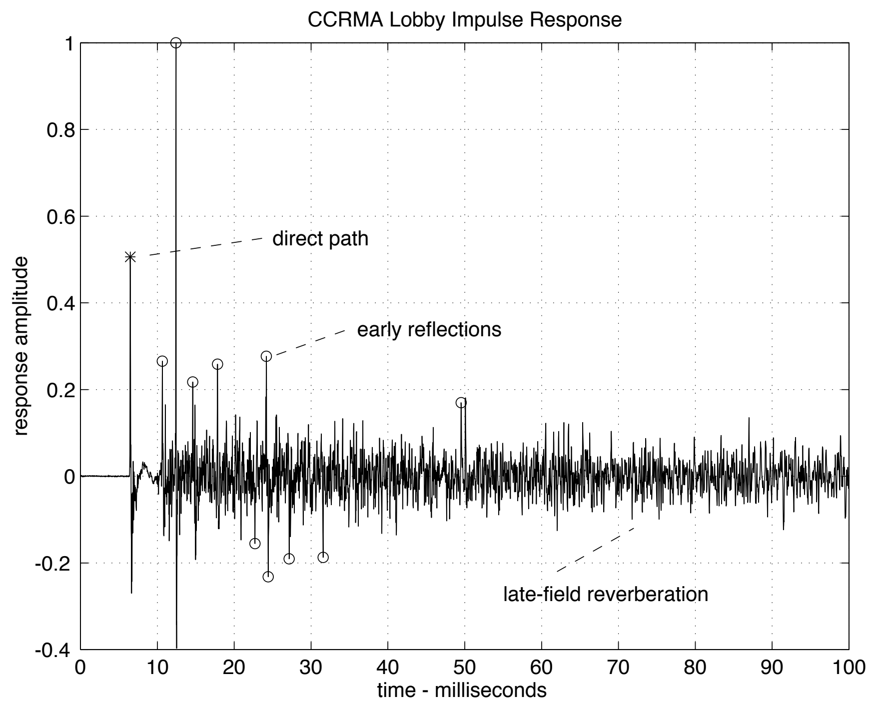
- Natural acoustic phenomenon that occurs when sound sources are played in a room
  - Thousands of echoes are generated as sound sources are reflected against wall, ceiling and floors
  - Reflected sounds are delayed, attenuated and low-pass filtered: high-frequency component decay faster
  - The patterns of myriads of echoes are determined by the volume and geometry of room and materials on the surfaces

Reverberation

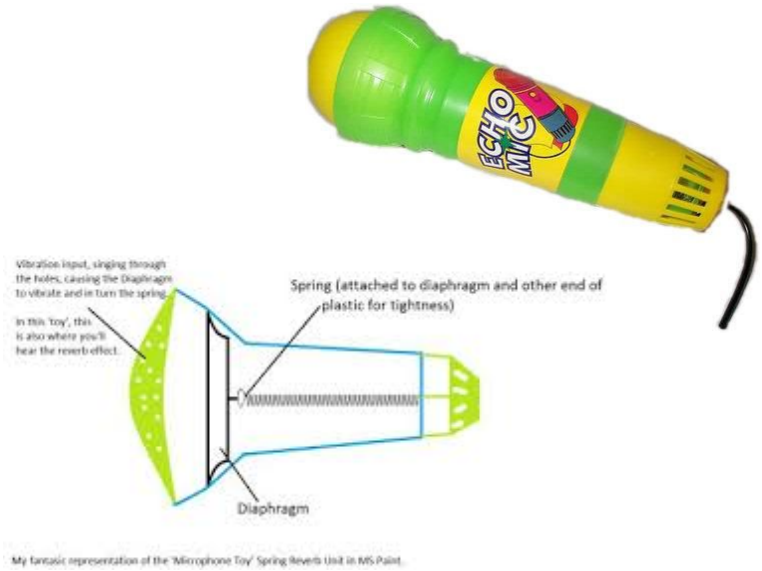
- Room reverberation is characterized by its impulse response
  - e.g. when a balloon pop is used as a sound source

- The room IR is composed of three parts
  - Direct path
  - Early reflections
  - Late-field reverberation

- RT60
  - The time that it takes the reverberation to decay by 60 dB from its peak amplitude



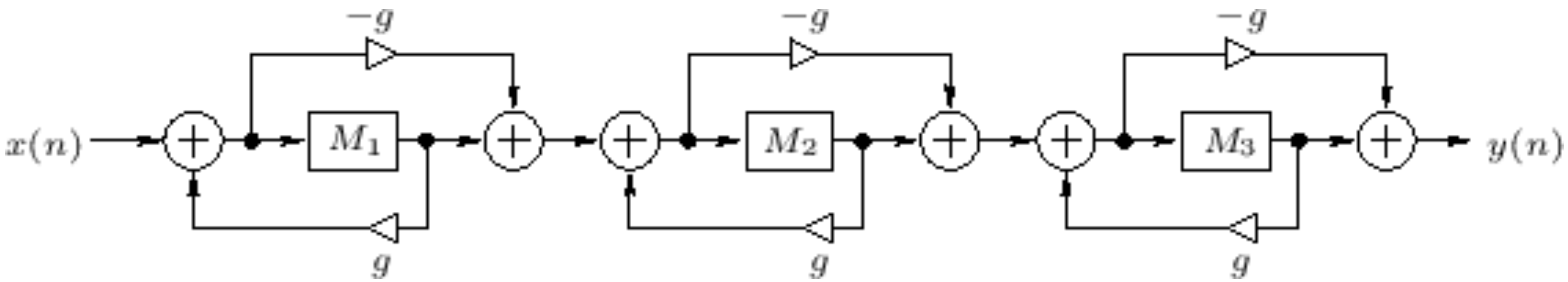
Artificial Reverberation



- Convolution reverb
  - Measure the impulse response of a room
  - Convolve input with the measured IR
- Mechanical reverb
  - Use metal plate and spring
  - EMT140 Plate Reverb: https://www.youtube.com/watch?v=HEmJpxCvp9M
- Delayline-based reverb
  - Early reflections: feed-forward delayline
  - Late-field reverb: allpass/comb filter, feedback delay networks (FDN)
  - “Programmable” reverberation

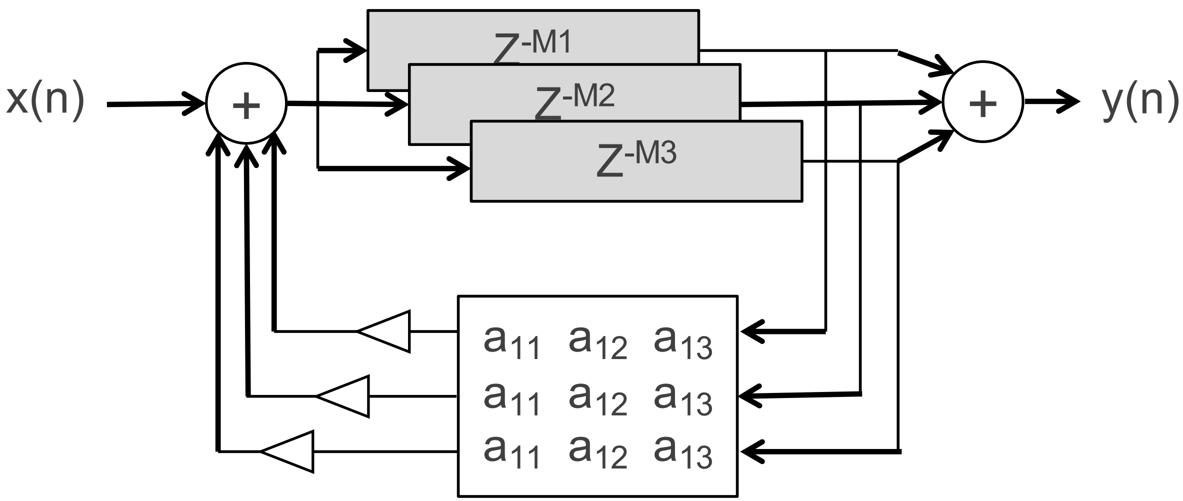
Delayline-based Reverb

- Schroeder model



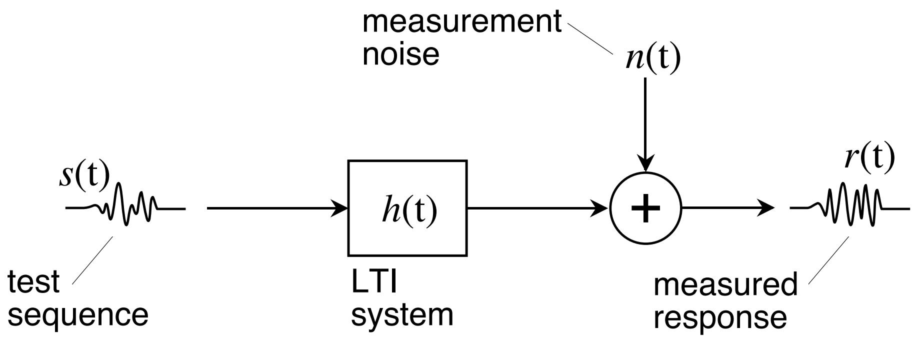
- Cascade of allpass-comb filters
- Mutually prime number for delay lengths

- Feedback Delay Networks



Convolution Reverb

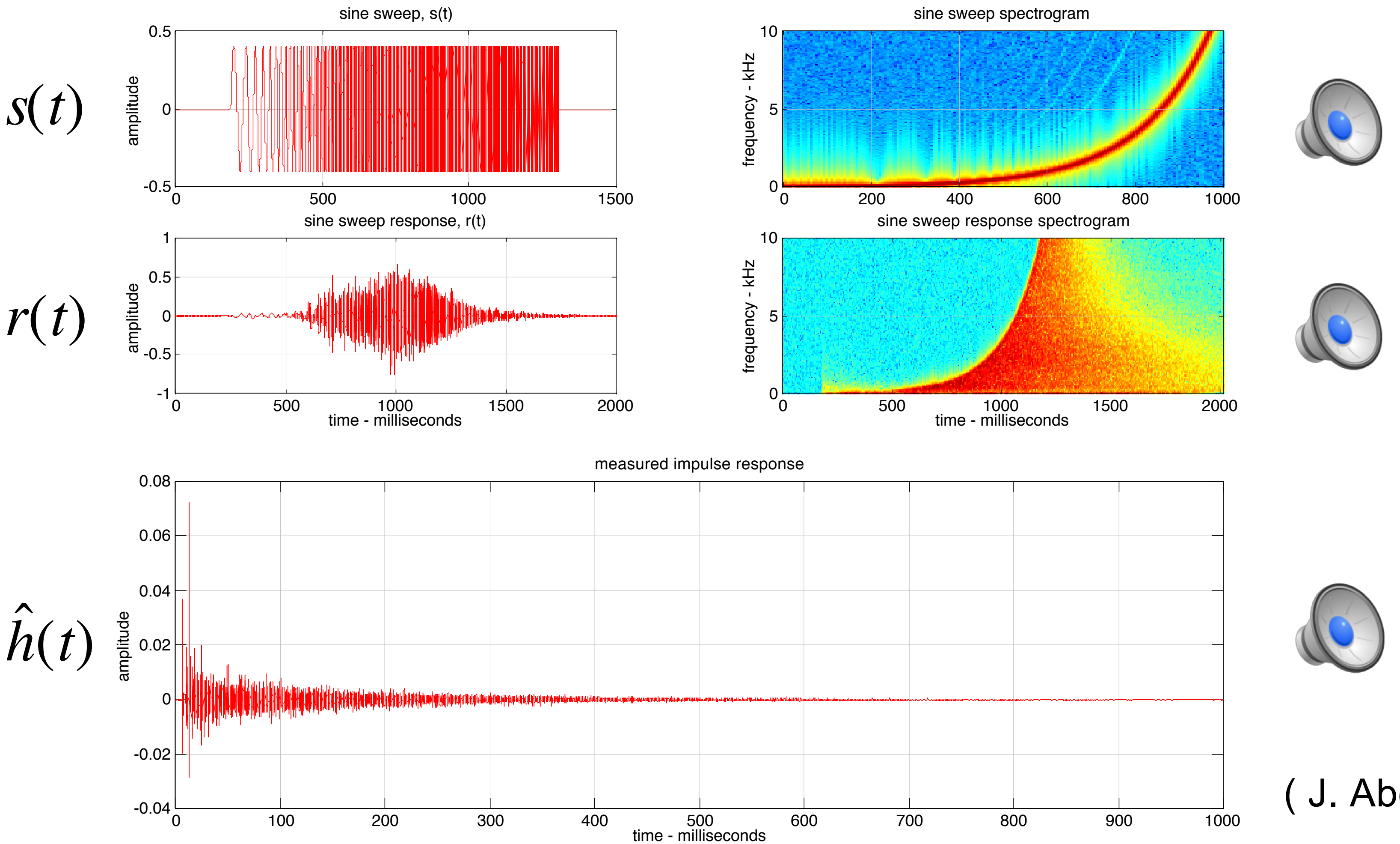
- Measuring impulse responses



$$r(t) = s(t) * h(t) + n(t)$$

- If the input is a unit impulse, SNR is low
- Instead, we use specially designed input signals
  - Golay code, allpass chirp or sine sweep: their magnitude responses are all flat but the signals are spread over time
- The impulse response is obtained using its inverse signal or inverse discrete Fourier transform

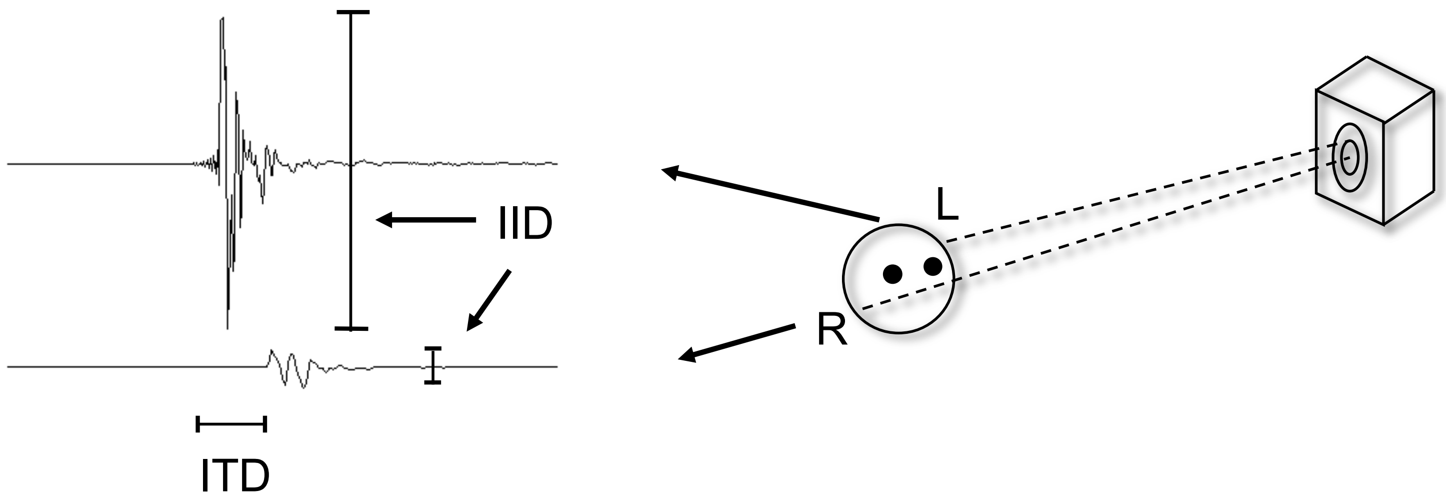
Convolution Reverb



(J. Abel)

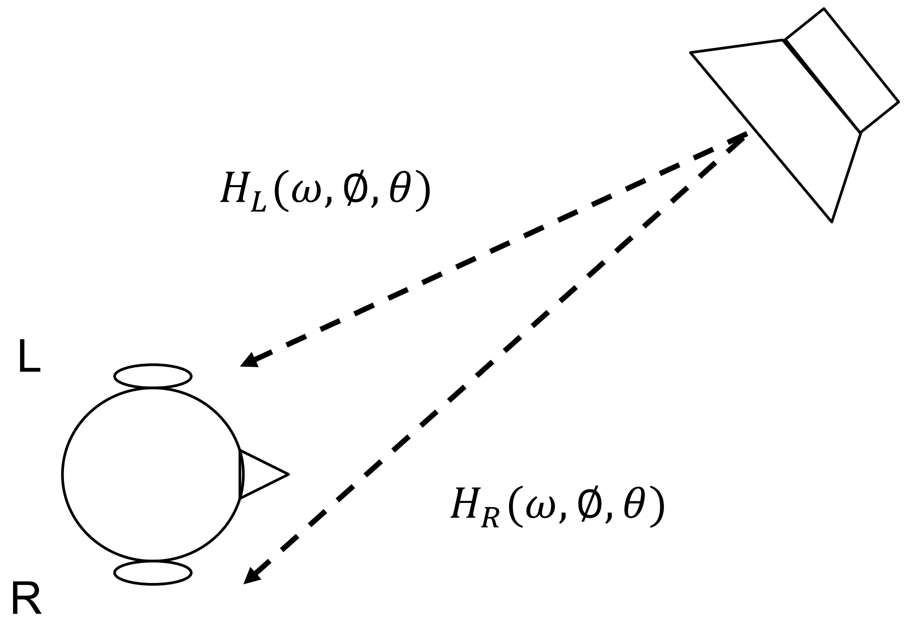
Spatial Hearing

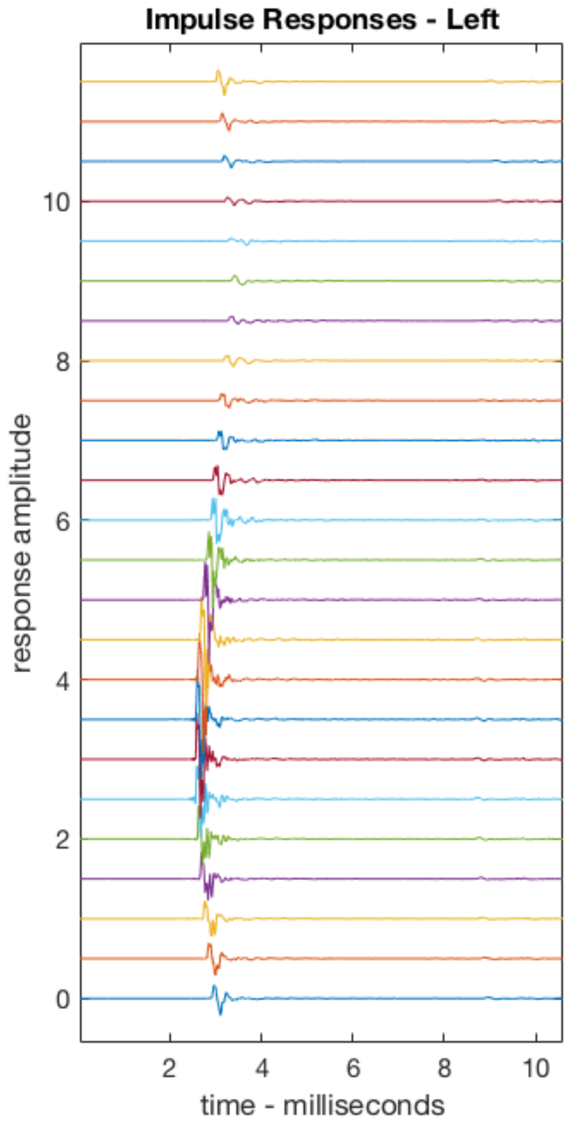
- A sound source arrives in the ears of a listener with differences in time and level
  - The differences are the main cues to identify where the source is.
  - We call them **ITD** (Inter-aural Time Difference) and **IID** (Inter-aural Intensity Difference)
  - ITD and IID are a function of the arrival angle.

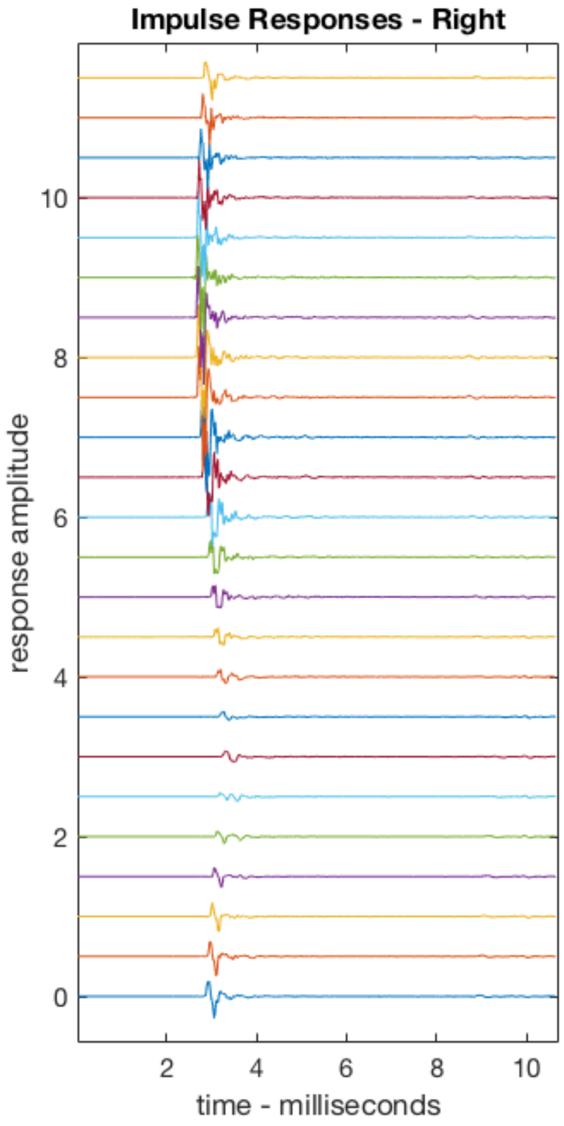


Head-Related Transfer Function (HRTF)

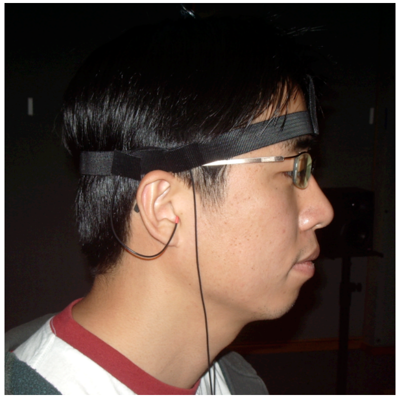
- A filter measured as the frequency response that characterizes how a sound source arrives in the outer end of ear canal
  - Determined by the refection on head, pinnae or other body parts
  - Function of azimuth (horizontal angle) and elevation (vertical angle)

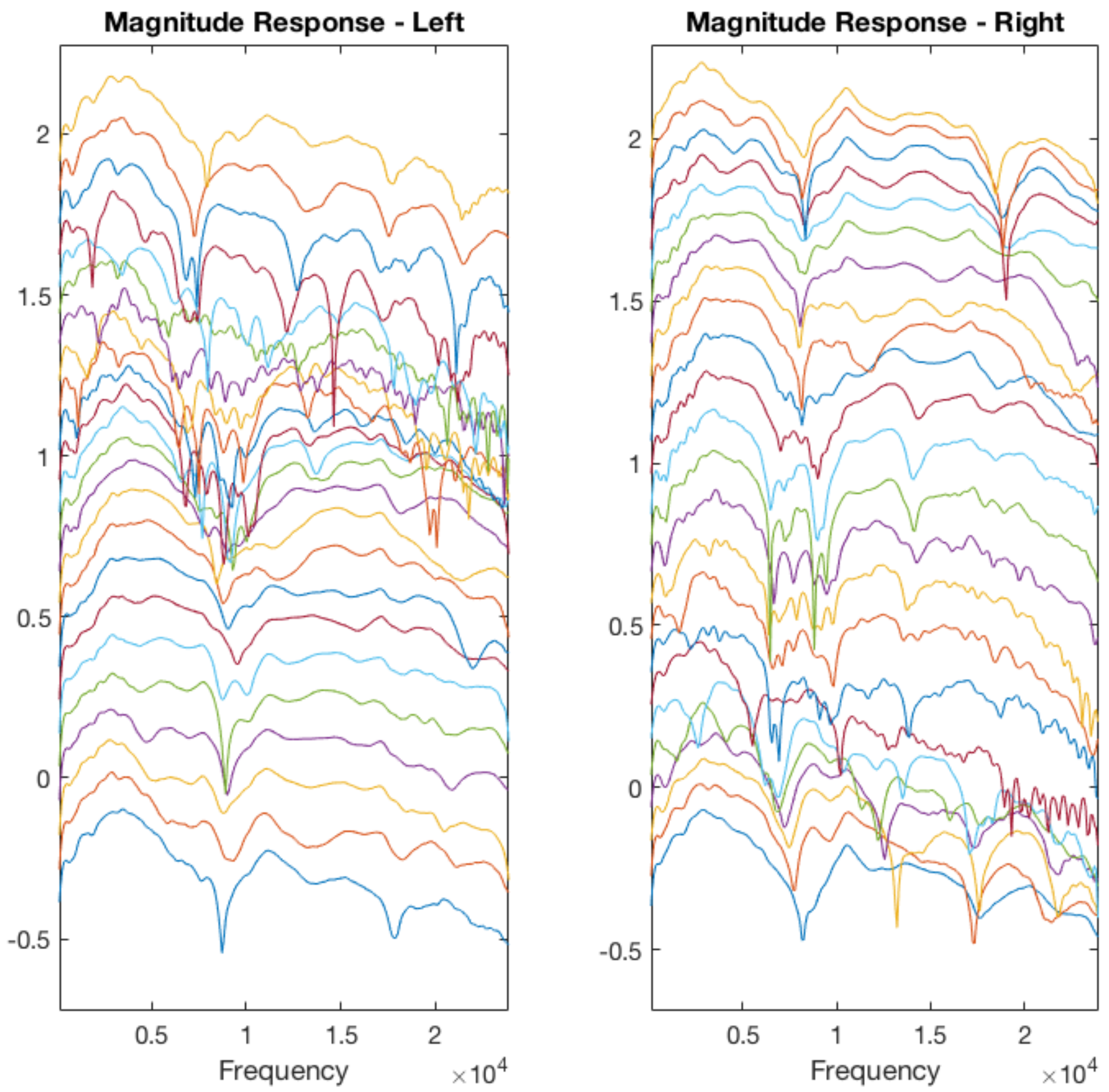






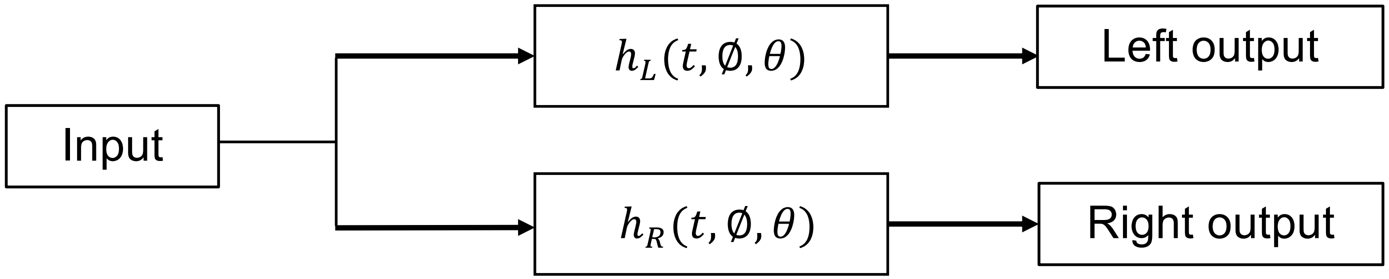
Measured Head-Related Impulse Responses (HRIR)





Magnitude
response
of the HRIRs

Binaural (3D Sound) Synthesis



- Rendering the spatial effect using the measured HRIRs as FIR filters
  - HRIRs are typically several hundreds sample long
  - Convolution or modeling by IIR filters

Web Audio Examples

- Pedal boards
  - https://pedals.io/

- 3D sounds
  - https://googlechrome.github.io/omnitone/#home